

Decentralized Local Energy Trading in Microgrids With Voltage Management

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Abstract—Local energy trading in microgrids is one of the emerging concepts in the area of distribution networks. A proper business model is required to manage local energy trading. The pricing mechanism is crucial because the agreed energy price determines the benefits of local energy trading. Designing a proper pricing mechanism with a specific objective considering the privacy of agents and respecting physical network constraints is a challenging task. This article proposes a decentralized algorithm for local energy trading in microgrids with an integrated pricing mechanism considering welfare maximization and network voltage management through local information exchange among neighbors. The proposed algorithm guarantees that the energy transactions do not violate voltage constraints in a physical network and agents' privacy is preserved. A two-stage approach is proposed to achieve fast convergence and increase the practicability of the algorithm. The simulation results are presented to verify the effectiveness of the proposed approach.

Index Terms—Decentralized optimization, energy pricing, information privacy, local energy trading, voltage management.

NOMENCLATURE

$\Delta \hat{p}_l$	Local estimation of average of Δp by agent l .
Δp	Total power mismatch.
η_l	Solution for agent l from distributed algorithm.
$\hat{\lambda}_l$	Local estimation of λ by agent l .
μ_l	Amount of power request of agent l to the other agents.
ρ	Step size.
k	Iteration counter for decentralized algorithm.
r	Iteration counter for voltage management algorithm.
$(\cdot)^k$	Corresponding parameter value at k th iteration.
$(\cdot)^r$	Corresponding parameter value at r th iteration.

β_l, θ_l	Utility function parameters of consumer $l \in \mathcal{C}$.
λ	Energy price in local energy market/dual variable.
\mathbf{I}	Identity matrix of appropriate dimensions.
\mathbf{p}_c	Vector of power demands.
\mathbf{p}_p	Vector of power generations.
\mathbf{P}	Matrix of active power flows.
\mathbf{p}	Vector of active power injection at nodes.
\mathbf{Q}	Matrix of reactive power flows.
\mathbf{q}	Vector of reactive power injection at nodes.
\mathcal{C}	Set of consumers.
\mathcal{E}	Set of lines.
\mathcal{N}	Set of all agents.
\mathcal{P}	Set of producers.
\mathcal{V}	Set of edges in communication topology.
Ω^l	Feasible region of p_l .
$\bar{v}_l, \underline{v}_l$	Available capacities of agent l to support other agents.
\bar{p}_l	Required amount of power need to inject at node l to clear voltage violation.
$\underline{p}_l, \bar{p}_l$	Lower and upper limit of p_l .
$\underline{v}_l, \bar{v}_l$	Lower and upper limit of voltage at node l .
$ \cdot $	Cardinality of a set.
a_l, b_l, c_l	Cost coefficients of producer $l \in \mathcal{P}$.
$C_l(\cdot)$	Cost function of producer $l \in \mathcal{P}$.
l, m	Indices of nodes/agents.
n	Index of lines.
n_c	Total number of consumers ($n_c \triangleq \mathcal{C} $).
n_p	Total number of producers ($n_p \triangleq \mathcal{P} $).
p_l	Power demand/generation of agent l .
$U_l(\cdot)$	Utility function of consumer $l \in \mathcal{C}$.
$v_{l,ref}$	Reference voltage at node l .
v_l	Voltage at node l .
$W_l(\cdot)$	Welfare function of agent l .

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I. INTRODUCTION

THE PROLIFERATION in the deployment of distributed energy resources (DERs) and flexible loads along with the digitization of the grids (smart grids) are shaping the future of the distribution networks [1]. The distribution network no longer exists as a passive unidirectional circuit supplying consumer loads when DERs are integrated into it. It becomes an active system having bidirectional power flow capability [2]. The integration of DERs into distribution networks has several advantages such as loss reduction, reduced dependency on the grid,

etc. However, the increasing penetration of DERs raises various operational and market challenges on the existing distribution networks such as voltage limit violations, line congestions, lack of visibility of DERs, intermittent energy imbalances, etc. [3]. These technical issues usually vary geographically or locally. Coordinated and controlled use of DERs could provide substantial benefits for the market as a whole, and commercial value to consumers, DERs owners, and distribution networks across the electricity sector. However, due to various technical, economic, communication, and regulatory barriers, it is challenging and, sometimes, impractical to design a centralized market system that serves several locations to solve such local problems from DERs integration. A centralized market limits opportunities for DERs to offer their benefits to broader electricity markets and networks, or to be rewarded for providing these benefits. Due to these reasons, existing incentives and tariffs (feed-in-tariff) designed to engage DERs in the energy trading are no longer attractive to motivate them to participate in the market. Thus, the design of the alternative energy market that supports active DERs participation has become important.

Microgrids provide one avenue for sharing and monetizing value from the increasing volume of DER through a local energy market. A microgrid is a part of a distribution network, which accommodates a variety of small-scale DERs and different types of energy users [4]. Several DERs and energy users serving in close proximity with defined geographical boundaries can be combined to form a microgrid. For simplicity, producers refer to DERs, and consumers refer to energy users, and they are jointly called agents. The agents in the microgrid can take the necessary actions to manage their consumption/generation. The producers can provide energy to the consumers locally, i.e., they can trade energy locally within the microgrid [5]. Local energy trading in microgrids is a viable option to handle local problems due to the integration of DERs into the distribution networks.

A local energy trading is one of the new concepts of growing importance in the area of distribution networks [6]. Local energy trading is beneficial to both producers and consumers compared to the trading with the utility grid. For local energy markets to become an attractive option for energy management in microgrids, a suitable business model with proper pricing mechanism or market clearing mechanism is necessary to facilitate the energy transactions among different parties [7], [8]. The pricing mechanism is crucial because an energy price determines the benefits of local energy trading and plays a significant role in shaping agents' behavior. The pricing mechanism should be computationally efficient and set with a defined objective of the trading. Agents in the market behave independently with their interest and have a set of private information, which they do not want to reveal to the public. Therefore, designing a proper pricing scheme maintaining privacy is a challenging task.

In contrast, a physical network is used for energy transfer; hence, excluding the physical network from local energy trading is not practical. In power networks, any local change in decisions of the users will result in global effects and cause external costs associated with it [9]. In distribution networks, node voltages are more sensitive to changes in active power due to the high value of resistance to reactance ratio of the lines [10]. Since the pricing

mechanism plays a vital role in changing the decision of agents, any change in energy price will change node voltages in the network and may also cause other external problems. Such voltage deviations can be managed using different voltage regulating devices, but may not be cost-effective for small-scale DERs and consumers. The pricing mechanism should be designed in such a way that agents' internal decisions will not create any external problems in the network and avoid the cost associated with it during the local energy trading. It makes the local energy trading concept more implementable. The pricing mechanism should be able to handle the external effects of the internal decisions of the agents. In a nutshell, designing a proper pricing mechanism with a specific objective considering the privacy of the agents and respecting physical network constraints is a major challenge in implementing the local energy trading in microgrids. The focus of this article is on the design of a proper pricing mechanism for local energy trading in microgrids with consideration of the privacy of the agents along with voltage management in the underlying power distribution network.

The rest of this article is organized as follows. In Section II, state of the art is presented, and major contributions are highlighted. Section III explains the model of the microgrid for local energy trading. In Section IV, decentralized approaches for energy pricing and voltage management are explained. Section V discussed the simulation results and, finally, Section VI concludes this article.

II. STATE OF THE ART

A. Related Works

Li *et al.* [11] presented a contract theoretic approach for energy trading between small-scale renewable energy suppliers and the aggregator. An aggregator acts as a coordinator of the energy trading process. An asymptotic Shapley value-based fair revenue division scheme is derived. A game-theoretic model for peer-to-peer (P2P) energy trading is proposed in [7] to maximize the social welfare of the buyers and sellers in a prosumer-based community microgrid. The centralized entity called P2P market operator is required to initiate the P2P energy trading. In [8], a hybrid energy market comprising an external utility company and a local trading market managed by a local trading center is investigated. The consumers and sellers adjust their behaviors to maximize their benefits after the local trading center sets the energy price. An incentive-compatible energy trading framework based on Stackelberg game is proposed in [12]. The electricity retailer acts as a mediator between energy users and shared energy storage.

A reinforcement learning-based indirect customer-to-customer energy trading in the distribution level is proposed in [13], where an energy broker facilitates the market operation and participants update strategies with learning capability. Park *et al.* [14] proposed an unsupervised learning algorithm for clustering of prosumers in local energy markets and auction mechanism for truthful electricity trading with local power exchange center as a broker.

A fog computing-based novel architecture for real-time vehicle-to-vehicle energy trading on the Internet of Vehicles

was proposed in [15]. Energy trading is controlled by a local market manager called the fog computing energy center (FCEC). The outcome of the trading depends on the interest of FCEC, whether it is profit- or nonprofit-driven, whereas vehicles are always profit-driven. A forward bilateral contract network for real-time P2P energy trading is proposed in [16], where suppliers act as intermediaries between sellers and buyers. A grid-influenced P2P energy trading is proposed in [17] to reduce the peak demand on centralized power system (CPS). A CPS decides the energy price which incentivizes the prosumers to participate in P2P energy trading to meet their demand and not to import any energy from the grid. Zhang *et al.* [18] proposed a contract game for direct energy trading between small-scale electricity suppliers and consumers. The energy management system for microgrids considering the physical network constraints such as node voltages, line-flow limits, etc., to ensure the effective operation of the microgrid was studied in [19]–[21].

The core objective in local energy trading is to use price signals to incentivize agents in the market to provide efficient generation and demand decisions by managing their flexibilities while taking into account the technical limitations of the network. Hence, the pricing mechanism should incorporate both economic incentives of agents linked to their welfare/cost/benefit as well as technical constraints linked to the physical limitation of the grid. The business model for local energy trading should consider both technical and economic aspects while designing the pricing mechanism. Most of the reviewed works [7], [8], [11]–[18], focused on the economic aspect for increasing welfare or decreasing costs, but they fail to consider the underlying electrical network for power distribution in the local energy trading model, and thus the outcome may not be feasible in practice. The network constraints should be considered in designing the trading mechanism since the physical network is used for actual energy transfer. However, limited research effort has been focused on the electrical constraints of the network, which is also a crucial requirement for any successful implementation of the local energy trading concept. The works in [19]–[21] consider network constraints in the efficient operation but ignore the energy pricing mechanism.

In addition, the works in [7], [8], and [11]–[18], considered a centralized coordinating agent to facilitate the local energy trading. The energy pricing and consideration of network constraints with a centralized coordinating entity are more straightforward as all the required information can be collected centrally. However, the presence of a centralized entity could endanger the privacy of the agents, as they have a set of private information and they may not be willing to reveal. Moreover, the centralized system has issues of scalability and computational burden with a large number of agents. Thus, there is a need to design a decentralized solution for efficient local energy trading in microgrids. As agents in the market behave independently to fulfill their interest, the individual optimal solution may not be socially optimal. Therefore, the pricing mechanism in local energy trading needs to be carefully designed such that individual optimal solutions align with the socially optimal solution, while

the privacy of the agent is preserved, and network constraints are respected.

To this end, there is a lack of a unified framework for decentralized energy pricing in local energy trading considering both technical and economic aspects together with the privacy of agents. This article aims to present a decentralized algorithm for local energy trading in microgrids with an integrated pricing mechanism considering welfare maximization and network voltage management through local information exchange among neighbors. The proposed method guarantees that the energy transactions do not violate voltage constraints in a physical network and agents' privacy is preserved.

B. Major Contributions

The major contributions are summarized as follows.

- 1) A local energy trading in microgrids is formulated as a social welfare maximization problem, and we design a fully decentralized algorithm, which neither requires any third party nor reveals any private information of the agents, to determine the energy trading price.
- 2) A fully decentralized approach is proposed to adjust the contribution of the agents to maintain node voltages within the acceptable ranges during local energy trading. A novel method to internalize the external effects of decision of agents on node voltages within the energy price so that the agents can trade in the local market without affecting the network performance is also proposed.
- 3) An important aspect of designing any iterative algorithm is the convergence speed. In this regard, a two-stage approach is proposed to achieve faster convergence while considering voltage management in the energy pricing mechanism. The two-stage approach improves the convergence speed and increases the practicability of the algorithm for real-time implementation.

III. SYSTEM MODEL

Consider a low-voltage microgrid with several DERs and responsive consumers who participate in local energy trading as producers and consumers. All DERs/consumer loads have electronic regulators to control the power generation/consumption level. Smart meters are installed at each consumer and DER locations. Additionally, each agent has a local workstation with an energy management system called a local energy management system (LEMS). The local energy trading algorithm is integrated with the LEMS software. The microgrid is divided into two layers: the physical layer and the virtual layer. The physical layer includes electrical connection of all players, whereas the virtual layer is the communication network that can be used for information exchange among agents. Smart meters are capable of communicating with other entities, and workstations are powerful enough to carry out the computational tasks. All the communication tasks are performed through the smart meters, and computations are performed in local workstations.

Consider a community microgrid with n_p producers and n_c consumers indexed by $1, 2, \dots, n_p$ and $n_p + 1, \dots, n_p + n_c$,

respectively. The producer set is denoted by \mathcal{P} and the consumer set is denoted by \mathcal{C} , and thus $\mathcal{N} = \mathcal{P} \cup \mathcal{C}$ and $N = n_p + n_c$. Assume that there is one agent at each node of the microgrid network, and indices of the agents are identical to the indices of the nodes. There are N nodes as there are N agents in the microgrid.

A. Consumer Agent Model

We consider a quadratic utility function [22] to quantify the usage benefit of consumer $l \in \mathcal{C}$ from consuming demand of p_l as follows:

$$U_l(p_l) = \begin{cases} \beta_l p_l - \theta_l p_l^2 & : 0 \leq p_l \leq \frac{\beta_l}{2\theta_l} \\ \frac{\beta_l^2}{4\theta_l} & : p_l \geq \frac{\beta_l}{2\theta_l} \end{cases}. \quad (1)$$

The utility $U_l(p_l)$ is the private information of consumer $l \in \mathcal{C}$ and not known to any other agents in the energy trading platform. The welfare function of the consumer $l \in \mathcal{C}$ is

$$W_l(p_l) = U_l(p_l) - \lambda p_l. \quad (2)$$

Each consumer $l \in \mathcal{C}$ chooses demand p_l to maximize its own welfare given by (2) subject to its demand constraints. The minimum demand represents the demand from *must run* appliances and also called baseline demand. Assume that flexible loads of the consumers are continuous. Consumers with flexible loads adjust their demand as a function of local energy trading platform price. Consumers with inflexible loads can be modeled by setting minimum and maximum demands equal to each other and the corresponding p_l becomes constant instead of the optimization variable [23].

B. Producer Agent Model

The cost function $C_l(p_l)$ of the DER owned by producer $l \in \mathcal{P}$ is usually approximated by a quadratic convex function of generated power p_l as

$$C_l(p_l) = a_l p_l^2 + b_l p_l + c_l. \quad (3)$$

The cost function $C_l(p_l)$ is the private information of producer $l \in \mathcal{P}$ and not known to any other agents in the energy trading platform. The welfare function of producer $l \in \mathcal{P}$ is modeled as

$$W_l(p_l) = \lambda p_l - C_l(p_l). \quad (4)$$

Each producer $l \in \mathcal{P}$ determines its generation p_l based on the energy trading platform price λ to maximize its welfare given by (4) subject to its generation constraints. If any producer has renewable-based DER such as solar PV, wind turbine, etc., with zero marginal cost, it can be modeled as *must take* generation by setting maximum and minimum generation limits equal to each other. The corresponding p_l variables become constant instead of optimization variable.

C. Microgrid Network Model

The underlying physical network of a community microgrid can be modeled as a radial system represented by the connected undirected graph/tree $\mathcal{G}(\{0, \mathcal{N}\}, \mathcal{E})$. We denote a line in $n \in \mathcal{E}$ by the pair (l, m) of nodes it connects. The incidence matrix

\mathbf{A}^0 of size $N \times (N + 1)$ is defined for graph \mathcal{G} . The entries are defined as $A_{nl}^0 = 1$ if line n leaves node l ; $A_{nl}^0 = -1$ if line n enters node l ; and $A_{nl}^0 = 0$, otherwise. Let \mathbf{a}_0 denote the first column of \mathbf{A}^0 and \mathbf{A} be the rest of \mathbf{A}^0 , i.e., $\mathbf{A}^0 = [\mathbf{a}_0 \ \mathbf{A}]$. The LinDistFlow model [24] can be summarized as

$$-\mathbf{A}^T \mathbf{P} = -\mathbf{p} \quad (5)$$

$$-\mathbf{A}^T \mathbf{Q} = -\mathbf{q} \quad (6)$$

$$\mathbf{a}_0 + \mathbf{A}\mathbf{v} = \mathbf{D}_r \mathbf{P} + \mathbf{D}_x \mathbf{Q} \quad (7)$$

where \mathbf{D}_r and \mathbf{D}_x are $N \times N$ diagonal matrices whose n th diagonal entries are constituted by r_{lm} and x_{lm} , respectively. Solving for \mathbf{P} and \mathbf{Q} and plugging them into (7) yields

$$\mathbf{v} = \mathbf{R}\mathbf{p} + \mathbf{X}\mathbf{q} + v_o \mathbf{1} \quad (8)$$

where $\mathbf{R} := \mathbf{A}^{-1} \mathbf{D}_r \mathbf{A}^{-T}$ and $\mathbf{X} := \mathbf{A}^{-1} \mathbf{D}_x \mathbf{A}^{-T}$. Define $\tilde{\mathbf{v}} = v_o \mathbf{1} + \mathbf{X}\mathbf{q}$ as a vector of reference voltages that include the substation voltage and the voltage contribution from the reactive power injections. This allows us to write the nodal voltage $\mathbf{v} \in \mathbb{R}^N$ as a linear function of the nodal power injection $\mathbf{p} \in \mathbb{R}^N$

$$\mathbf{v} = \tilde{\mathbf{v}} + \mathbf{R}\mathbf{p}. \quad (9)$$

A matrix $\mathbf{R} \in \mathbb{R}^{N \times N}$ depends on the topology of the network and the line resistances [25].

D. Network Communication Model

An undirected connected graph $\mathcal{G}_c = \{\mathcal{N}, \mathcal{V}\}$ is used to represent the communication topology of the network. A edge between nodes l and m is denoted by a pair $(l, m) \in \mathcal{V}$, which means agents l and m can exchange information with each other. We assume that self-loops are not allowed, i.e., $(l, l) \notin \mathcal{V}$ and the graph is balanced. The neighbor set of node (agent) l is defined as $\mathcal{N}_l = \{m \in \mathcal{N} | (l, m) \in \mathcal{V}\}$. We assume that nodes can communicate bidirectionally with their physically connected neighbors. Hence, the topology of the communication network is same as that of the power network. However, it is allowed to have a different communication network topology than the topology of the power network. A doubly stochastic matrix $\mathbf{E} = (e_{lm})_{N \times N}$ associated with \mathcal{G}_c is defined as follows:

$$e_{lm} = \begin{cases} \frac{1}{\max(|\mathcal{N}_l|, |\mathcal{N}_m|) + 1} & : m \in \mathcal{N}_l \\ 1 - \sum_{m \in \mathcal{N}_l} \frac{1}{\max(|\mathcal{N}_l|, |\mathcal{N}_m|) + 1} & : l = m \\ 0 & : \text{otherwise} \end{cases}. \quad (10)$$

IV. ENERGY PRICING AND VOLTAGE MANAGEMENT

A. Energy Pricing Mechanism

The energy price depends on the desired objective of local energy trading. In this article, the goal is to maximize the social welfare of the microgrid given by

$$W(\mathbf{p}_p, \mathbf{p}_c) \triangleq \sum_{l \in \mathcal{P}} W_l(p_l) + \sum_{l \in \mathcal{C}} W_l(p_l). \quad (11)$$

Mathematically

$$\arg \max_{\mathbf{p}_p, \mathbf{p}_c} W(\mathbf{p}_p, \mathbf{p}_c)$$

$$\text{s.t. } p_l \leq p_l \leq \bar{p}_l \quad (12)$$

where $\mathbf{p}_p = [p_1, p_2, \dots, p_{n_p}]^T$ and $\mathbf{p}_c = [p_1, p_2, \dots, p_{n_c}]^T$. The optimization problem (12) can be reformulated as a convex problem as

$$\begin{aligned} \underset{\mathbf{p}_p, \mathbf{p}_c}{\text{argmin}} \quad & \left[\sum_{l \in \mathcal{P}} C_l(p_l) - \sum_{l \in \mathcal{C}} U_l(p_l) \right] \\ \text{s.t. } \quad & \mathbf{A}_p \mathbf{p}_p + \mathbf{A}_c \mathbf{p}_c = 0 \\ & p_l \leq p_l \leq \bar{p}_l \quad \forall l \in \mathcal{N} \end{aligned} \quad (13)$$

where $\mathbf{A}_p = [-1, -1, \dots, -1]_{1 \times n_p}$, $\mathbf{A}_c = [1, 1, \dots, 1]_{1 \times n_c}$. The Lagrangian function corresponding to (13) is

$$\mathcal{L} = \sum_{l \in \mathcal{P}} C_l(p_l) - \sum_{l \in \mathcal{C}} U_l(p_l) + \lambda (\mathbf{A}_p \mathbf{p}_p + \mathbf{A}_c \mathbf{p}_c). \quad (14)$$

The dual variable λ corresponding to the global equality constraint in (13) represents the price of energy in the local energy market. The inequality constraints are not included in (14) as they are local constraints and can be treated as the boundaries of the feasible region of the local problems.

Solving (13) in a centralized fashion violates the privacy of the agents because the centralized coordinator needs complete information all agents. It should be solved with minimal information exchange between the agents. On the basis of the principle of dual decomposition [26], a distributed iterative approach for solving (13) can be defined as

$$\lambda^k = [\lambda^{k-1} + \rho \Delta p^{k-1}]^+, \Delta p^{k-1} = \mathbf{A}_p \mathbf{p}_p^{k-1} + \mathbf{A}_c \mathbf{p}_c^{k-1} \quad (15)$$

$$p_l^k = \begin{cases} \arg \min_{p_l \in \Omega^l} [C_l(p_l) - \lambda^k p_l] & : l \in \mathcal{P} \\ \arg \min_{p_l \in \Omega^l} [\lambda^k p_l - U_l(p_l)] & : l \in \mathcal{C} \end{cases} \quad (16)$$

where $[\cdot]^+$ means $\max(0, \cdot)$.

Each agent $l \in \mathcal{N}$ can solve (16) in parallel locally. However, the local optimizations in (16) require knowledge of the global (i.e., dual) variable λ . Updating the global variable λ using (15) requires the knowledge of total power mismatch Δp . The coupling between local problems and global variables in an iterative approach still requires a central coordinating agent, which is undesirable. Inspired by Olfati-Saber *et al.*'s study [27], we propose a fully decentralized algorithm for local energy trading to overcome this issue. The proposed algorithm requires information exchange among neighboring agents only and constitutes a decentralized solution.

The updating rules for agents' coordination are as follows:

$$\forall l \in \mathcal{N} : \hat{\lambda}_l^k = \left[\sum_{m \in \mathcal{N}} e_{lm} \hat{\lambda}_m^{k-1} + \rho \Delta \hat{p}_l^{k-1} \right]^+ \quad (17)$$

$$p_l^k = \begin{cases} \arg \min_{p_l \in \Omega^l} [C_l(p_l) - \hat{\lambda}_l^k p_l] & : l \in \mathcal{P} \\ \arg \min_{p_l \in \Omega^l} [\hat{\lambda}_l^k p_l - U_l(p_l)] & : l \in \mathcal{C} \end{cases} \quad (18)$$

$$\Delta \hat{p}_l^k = \begin{cases} \sum_{m \in \mathcal{N}} e_{lm} \Delta \hat{p}_m^{k-1} + p_l^{k-1} - p_l^k & : l \in \mathcal{P} \\ \sum_{m \in \mathcal{N}} e_{lm} \Delta \hat{p}_m^{k-1} + p_l^k - p_l^{k-1} & : l \in \mathcal{C} \end{cases} \quad (19)$$

Algorithm 1: Decentralized Algorithm for Price Update.

- Require:** Step-size and tolerance levels for termination
Ensure: Energy price for local energy trading λ^* Each agent $l \in \mathcal{N}$
- 1: Initializes all $\hat{\lambda}_l$, p_l , and $\Delta \hat{p}_l$. Sets the tolerance levels ϵ_p and ϵ_λ for the power mismatch and price, respectively. Set step size ρ ($0 < \rho < 1$)
 - 2: **repeat**
 - 3: Broadcasts $\hat{\lambda}_l^{k-1}$ and $\Delta \hat{p}_l^{k-1}$ to neighbors and receives neighbors' information;
 - 4: Updates $\hat{\lambda}_l^k$ according to (17);
 - 5: Updates p_l^k according to (18);
 - 6: Updates $\Delta \hat{p}_l^k$ according to (19);
 - 7: **until** $\Delta \hat{p}_l$ converges as $|\Delta \hat{p}_l^k| < \epsilon_p, \forall l \in \mathcal{N}$ and λ converges as $|\hat{\lambda}_l^k - \hat{\lambda}_l^{k-1}| < \epsilon_\lambda, \forall l \in \mathcal{N}$;
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By comparing welfare definitions in Section III with (18), $\hat{\lambda}_l$ can be interpreted as the local prices of energy offered to agents, which are used for energy transactions. These are the prices that the producers are paid for providing energy, and consumers pay for consuming energy. Since $\hat{\lambda}_l$ are the local estimations of the global dual variable λ , all $\hat{\lambda}_l$ throughout the network should converge to a unique value at the optimal point, which gives the energy price in the local energy market. Once the local energy prices equate with each other, the power mismatch becomes zero, and the optimal solution is achieved. Based on the local energy price $\hat{\lambda}_l$, each agent updates its generation/consumption level greedily according to (18). If a particular agent solves the optimization problem independently in a selfish way, the components that depend on the private information of other agents in the iteration step are not available. The feasibility region is also reduced for a particular agent only. Each agent uses its private information and power mismatch estimation from neighbors to update its state, which ensures the privacy of the agents in the market. For sufficiently small positive value of ρ , the algorithm is stable and all the variables converge, i.e., $\hat{\lambda}_l^k \rightarrow \lambda^*$, $p_l^k \rightarrow p_l^*$, $\Delta \hat{p}_l^k \rightarrow 0$, as $k \rightarrow \infty$, and $\forall l \in \mathcal{N}$. The convergence characteristics of the proposed algorithm are discussed in Appendix A. The process for algorithm price update is summarized in Algorithm 1.

B. Voltage Management in Local Energy Trading

Let \mathbf{v}^k and \mathbf{p}^k denote the vector of node voltage magnitudes and agents' decision on active power injections at k th iteration of the decentralized algorithm, respectively. If $\Delta \bar{\mathbf{p}}^k = \mathbf{p}^k - \mathbf{p}^{k-1}$ is the vector describing small variations in active power injections at nodes between iterations $k-1$ and k , then based on (9), the vector of corresponding variations in node voltage magnitudes $\Delta \mathbf{v}^k = \mathbf{v}^k - \mathbf{v}^{k-1}$ can be expressed as

$$\Delta \mathbf{v}^k = \mathbf{R} \Delta \bar{\mathbf{p}}^k. \quad (20)$$

The relation that describes how the bus voltage magnitudes evolve with the change in active power injections in successive

iterations is given by

$$\mathbf{v}^k = \mathbf{v}^{k-1} + \mathbf{R} \Delta \tilde{\mathbf{p}}^k. \quad (21)$$

The active power deviations $\Delta \tilde{\mathbf{p}}$ required to bring the voltage deviations of $\Delta \mathbf{v}$ is given as

$$\Delta \tilde{\mathbf{p}} = \mathbf{R}^{-1} \Delta \mathbf{v}. \quad (22)$$

The aim is to maintain each node voltage magnitudes within the acceptable ranges as

$$\underline{v}_l \leq v_l \leq \bar{v}_l. \quad (23)$$

The agent at node l sets the reference voltage at the k th iteration as

$$v_{l,\text{ref}}^k = \begin{cases} \underline{v}_l & : v_l^k < \underline{v}_l \\ v_l^k & : \underline{v}_l \leq v_l^k \leq \bar{v}_l \\ \bar{v}_l & : v_l^k > \bar{v}_l \end{cases}. \quad (24)$$

The voltage violation at node l from its reference value at the k th iteration is given by

$$\vartheta_l^k = v_{l,\text{ref}}^k - v_l^k. \quad (25)$$

The agent l estimates the additional amount of active power needed to clear the violation given by (25) as

$$\varrho_l^k = \frac{\alpha \vartheta_l^k}{R_{ll}} \quad (26)$$

where $\alpha > 0$ is some constant. The value of α should be chosen carefully to ensure the stability of the algorithm. The total estimated active power to be injected by the agent at node l to clear the voltage violation is

$$\tilde{p}_l^k = p_l^k + \varrho_l^k. \quad (27)$$

If $\underline{p}_l \leq \tilde{p}_l^k \leq \bar{p}_l, \forall l \in \mathcal{N}$, every agent can provide the estimated amount of active power to clear the voltage violation at its node by itself. However, if $\tilde{p}_l^k > \bar{p}_l$ or $\tilde{p}_l^k < \underline{p}_l$, agent l cannot provide the estimated amount of active power to clear the voltage violation at its node by itself. If there is at least one agent that cannot clear the violation by itself, other agents having additional capacity will decide an extra amount of power they can provide to support other agents through a distributed algorithm.

Each agent shares a set of variables μ_l , \bar{v}_l , and \underline{v}_l with its neighbors with the initial values set as follows:

$$\begin{aligned} \mu_l^0 &= \begin{cases} \tilde{p}_l^k - \bar{p}_l & : \tilde{p}_l^k > \bar{p}_l \\ \tilde{p}_l^k - \underline{p}_l & : \tilde{p}_l^k < \underline{p}_l \\ 0 & : \text{otherwise} \end{cases} \\ \bar{v}_l^0 &= \begin{cases} \bar{p}_l - \tilde{p}_l^k & : \tilde{p}_l^k < \bar{p}_l \\ 0 & : \text{otherwise} \end{cases} \\ \underline{v}_l^0 &= \begin{cases} \underline{p}_l - \tilde{p}_l^k & : \tilde{p}_l^k > \underline{p}_l \\ 0 & : \text{otherwise} \end{cases}. \end{aligned} \quad (28)$$

Each agent updates the corresponding variables based on its own and the exchanged information with its neighbors. The updating

Algorithm 2: Algorithm for Voltage Management.

Require: Node voltages: $v_1^k, \dots, v_l^k, \dots, v_M^k$

Ensure: Contribution for voltage management η
Each agent $l \in \mathcal{N}$

- 1: Sets the tolerance level ϵ ;
 - 2: Sets reference node voltage $v_{l,\text{ref}}^k$ according to (24);
 - 3: Computes the voltage violation at node ϑ_l^k using (25);
 - 4: Estimates the additional power needed to clear the violation ϱ_l^k by (26) and total estimated power \tilde{p}_l^k by (27);
 - 5: Initializes the parameters of the distributed algorithm μ_l , \underline{v}_l , and \bar{v}_l according to (28);
 - 6: **repeat**
 - 7: Broadcasts the parameters μ_l^r , \underline{v}_l^r , and \bar{v}_l^r to neighbors;
 - 8: Updates parameters according to (29) after receiving neighbors' information;
 - 9: Calculates contribution η_l^r according to (30);
 - 10: **until** contribution parameter η_l^r converges to (31) as $|\eta_l^r - \eta_l^{r-1}| < \epsilon$;
-

rules for variables are as follows:

$$\begin{aligned} \mu_l^r &= \sum_{m \in \mathcal{N}} e_{lm} \mu_m^{r-1} \\ \bar{v}_l^r &= \sum_{m \in \mathcal{N}} e_{lm} \bar{v}_m^{r-1} \\ \underline{v}_l^r &= \sum_{m \in \mathcal{N}} e_{lm} \underline{v}_m^{r-1}. \end{aligned} \quad (29)$$

Then, at each iteration r , agent l with $\underline{v}_l^r \neq 0$ or $\bar{v}_l^r \neq 0$ calculates its contribution η_l^r as

$$\eta_l^r = \begin{cases} \frac{\mu_l^r}{\underline{v}_l^r} \underline{v}_l^0 & : \mu_l^r < 0 \\ \frac{\mu_l^r}{\bar{v}_l^r} \bar{v}_l^0 & : \mu_l^r \geq 0 \end{cases} \quad (30)$$

where η_l^r will asymptotically converge to

$$\eta_l = \begin{cases} \left(\frac{\sum_{l=1}^N \mu_l^0}{\sum_{l=1}^N \underline{v}_l^0} \right) \underline{v}_l^0 & : \lim_{r \rightarrow \infty} \mu_l^r < 0 \\ \left(\frac{\sum_{l=1}^N \mu_l^0}{\sum_{l=1}^N \bar{v}_l^0} \right) \bar{v}_l^0 & : \lim_{r \rightarrow \infty} \mu_l^r \geq 0. \end{cases} \quad (31)$$

The details behind this result are given in Appendix B. Once the algorithm is converged, agent l updates its power generation/consumption decision to

$$p_l^k = \begin{cases} \bar{p}_l & : \tilde{p}_l^k + \eta_l > \bar{p}_l \\ \underline{p}_l & : \tilde{p}_l^k + \eta_l < \underline{p}_l \\ \tilde{p}_l^k + \eta_l & : \text{otherwise} \end{cases}. \quad (32)$$

The algorithm for voltage management in local energy trading is given in Algorithm 2.

C. Implementation Procedure

In real applications, it is not practical to change the energy price λ continuously in the electricity market. It is more practical and feasible to update the energy price λ in a fixed period. Thus, the proposed energy trading algorithm needs to be run at the beginning of each period and the outcome is kept constant throughout that period.

The local energy trading algorithm proposed in this article uses an iterative approach through the interaction among agents. The responses from agents would change in response to the price, and this change will, in turn, affects the energy price. The algorithm starts with the initialization of the required variables by each agent l and the broadcast of initial values to other agents. The selection of initial values has a significant influence on the convergence of the algorithm. As the cost/utility function parameters and generation/demand limits do not change abruptly from one period to another period, it is reasonable to consider stable solution of the previous time period as initial values for the next period. Upon receiving the latest update from neighboring agents, each agent l updates local price $\hat{\lambda}_l^k$ according to (17) and calculates the optimal energy generation/demand, p_l^k using (18) based on the recently updated local price and then updates the local estimate of average global power mismatch $\Delta\hat{p}_l^k$ according to (19).

If we consider the network voltage management in the energy pricing, the sequence of execution is slightly different from the one explained before. An intermediate step will take place between calculating p_l^k using (18) and updating $\Delta\hat{p}_l^k$ using (19) by each agent. Each agent l executes voltage management algorithm given in Algorithm 2 before calculating $\Delta\hat{p}_l^k$ after it calculates p_l^k . The value of p_l^k used for updating $\Delta\hat{p}_l^k$ in this case is not the same as given by (18). Each agent l calculates its contribution for voltage management η_l from Algorithm 2 and updates p_l^k according to (32). Then, $\Delta\hat{p}_l^k$ is calculated using the latest updated value of p_l^k . Assume that each agent has the complete knowledge of the network topology, and real-time voltage measurement is available. In this article, we use the concept of virtual measurement using the power flow engine for simulation purpose [28]. After each iteration, the stopping criteria are examined. If $|\Delta\hat{p}_l^k|$ and $|\hat{\lambda}_l^k - \hat{\lambda}_l^{k-1}|$ are lower than ϵ_p and ϵ_λ , respectively, and there is no voltage violation, the algorithm iterations are stopped. Otherwise, the process is repeated until the stopping criteria is fulfilled. The overall flow of the implementation procedure is summarized in Fig. 1

V. RESULTS AND DISCUSSIONS

This section presents numerical case studies to demonstrate the virtue of the proposed local energy trading mechanism. We consider a microgrid with 16 producers and 16 consumers as shown in Fig. 2. The microgrid has the same network structure as that of the IEEE 33-bus distribution system. Each agent can communicate with its physically connected neighbors. The simulation results are based on the assumption of perfect and fast bidirectional communication and real-time measurements are available. There are five solar PV systems in the microgrid at nodes 1, 2, 19, 30, and 32, which are nondispatchable

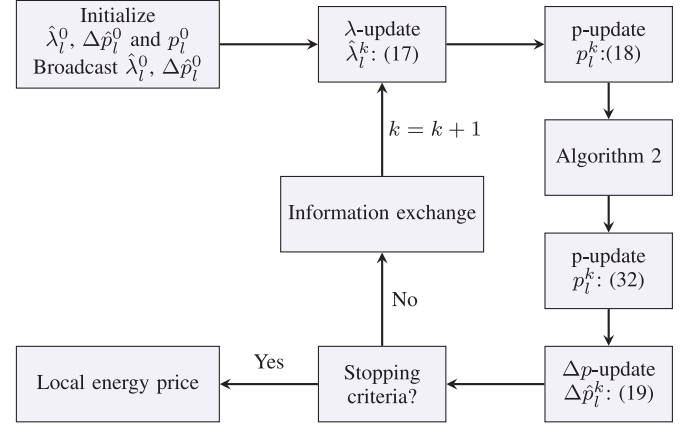


Fig. 1. Flowchart for decentralized local energy trading.

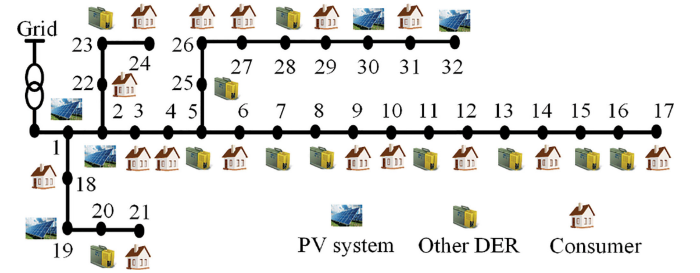


Fig. 2. 33-bus microgrid network.

generation. We consider only one period in this case study.¹ The minimum and maximum generation for producers are in the range of $[0, 3]$ and $[3, 8]$ kW, respectively. Similarly, minimum and maximum demands for consumers are assumed to be in the range of $[0, 3]$ and $[3, 8]$ kW, respectively. The utility function parameters of consumers β_l and θ_l are considered to be $\beta \in [10, 20]$ $\text{\$/kWh}$ and $\theta \in (0, 1]$ $\text{\$/kWh}^2$, respectively, whereas the cost function parameters for producers a_l and b_l are selected randomly from the interval $(0, 1]$ $\text{\$/kWh}^2$ and $[0, 10]$ $\text{\$/kWh}$, respectively, and c_l is set to 0. The required tolerances for termination are set to $\epsilon_\lambda = 0.001$ and $\epsilon_p = 0.001$. We assume that the voltage magnitudes must lie within $\pm 5\%$ of 1 p.u. at all nodes. On the basis of the data and the proposed algorithm, all the simulations are conducted in the MATLAB 2016a environment with an Intel Xeon CPU E5-1630 v4@3.70 GHz, 16 GB RAM. The results in different cases are discussed in the following sections. The initial values are set as $\hat{\lambda}_l^0 = C_l'(p_l)$, $\forall l \in \mathcal{P}$; $\hat{\lambda}_l^0 = U_l'(\bar{p}_l)$, $\forall l \in \mathcal{C}$ and $\Delta\hat{p}_l^0 = 0$.

A. Without Voltage Management

Fig. 3 shows the evolution of total demand and supply during local energy trading using the proposed decentralized algorithm without considering the network voltage management. In Fig. 3,

¹As the multiperiod problem can be easily decoupled into multiple problems at a separate period using relaxation techniques, we solve the problem for a single period to demonstrate the performance of the proposed method in a better way. The multiperiod study is beyond the scope of this article.

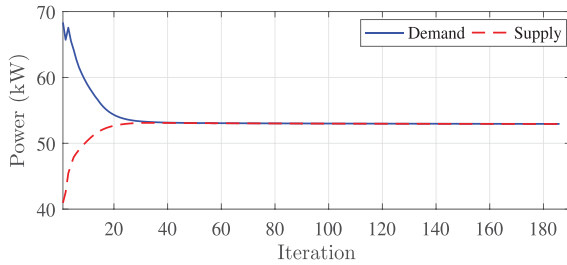


Fig. 3. Evolution of total demand and supply without voltage management.

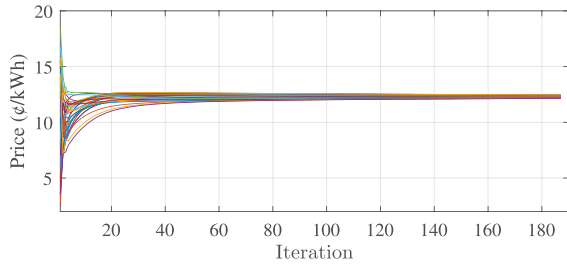


Fig. 4. Evolution of local prices without voltage management.

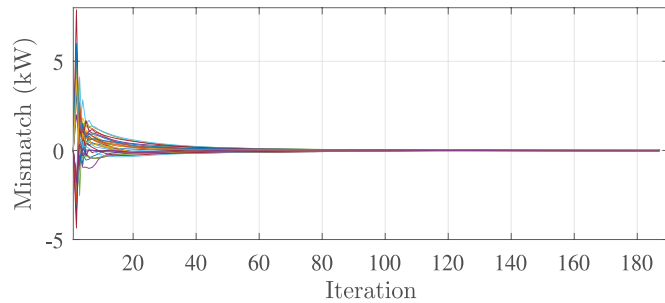


Fig. 5. Evolution of estimation of power mismatch without voltage management.

there exists a mismatch between the total demand and supply initially. The evolution of local energy prices $\hat{\lambda}_l$ is shown in Fig. 4 and the evolution of local estimation of mismatch power $\Delta\hat{p}_l$ is shown in Fig. 5. As the algorithm proceeds, each local price converges toward common value λ^* and all the local estimation of power mismatch $\Delta\hat{p}_l, \forall l \in \mathcal{N}$ get close to zero as shown in Fig. 5. The total power imbalance Δp goes to zero, as shown in Fig. 7 as the local estimation of power mismatches get close to zero. The total supply meets the total demand, and thus the power balance condition given in (13) is satisfied gradually, as shown in Fig. 3. The proposed decentralized algorithm satisfies the termination conditions $|\hat{\lambda}_l^k - \hat{\lambda}_l^{k-1}| < \epsilon_\lambda$ and $|\Delta\hat{p}_l| < \epsilon_p$ when number of iterations $k = 187$ and the results converge. The voltage magnitudes at some nodes are less than 0.95 p.u. and violate the safe operating limits, as shown in Fig. 8 when the voltage management strategy is not considered in the energy pricing mechanism. In real-time applications, it may not be practical to run the algorithm for a large number of iterations to reach a very precise solution. Thus, the algorithm should be stopped

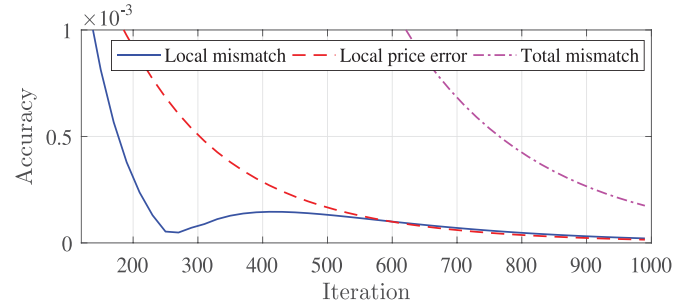


Fig. 6. Accuracy versus iterations without voltage management.

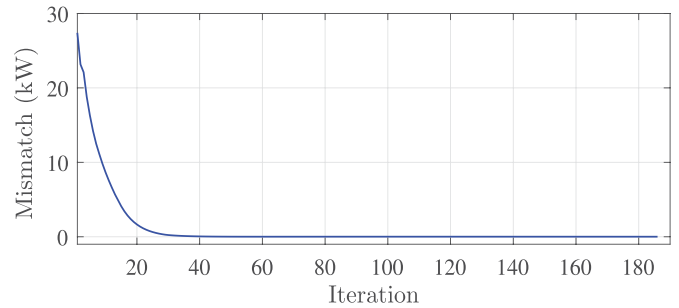


Fig. 7. Evolution of power imbalance Δp without voltage management.

after a certain number of iterations. The compromise between time and accuracy is necessary. Fig. 6 shows the variation of error in local power mismatches, local prices, and global power mismatch with the number of iterations. The required number of iterations depends on the accuracy required.

B. With Voltage Management

Fig. 9 shows the evolution of total supply and demand in the local energy market while incorporating the voltage management strategy within the energy pricing mechanism. The effect of considering the network voltages in the energy pricing mechanism can be seen in Fig. 9. The evolution of supply and demand curves in Fig. 9 show different nature compared with the evolution curves without voltage management in Fig. 3. The decision of agents depends only on the energy price in the previous case without voltage management. However, with voltage management, agents' decision depends on both the energy price and node voltage magnitudes. Unlike the case without voltage management shown in Fig. 7, with the same initial conditions, the imbalance between the supply and demand with voltage management shown in Fig. 10 has different characteristics. It can be observed in Fig. 10 that spikes occur at several places as the algorithm proceeds. These spikes arise from the fact that voltage limits violations force the agents to change their decision abruptly from one iteration to another iteration to bring back the voltages in the acceptable range. It results in a change in the magnitude of power imbalance. Such changes in agents' decision on demand/generation due to voltage violations in the current iteration will be reflected in energy price in the next iteration. The termination conditions are satisfied when $k = 277$,

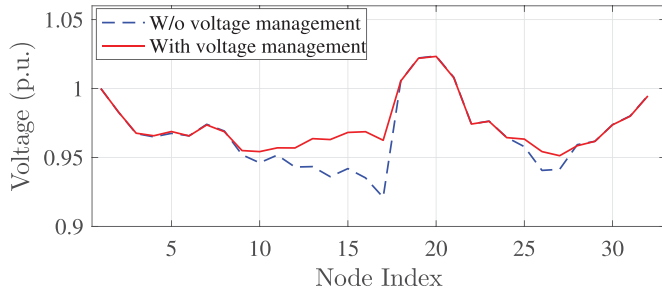


Fig. 8. Node voltages with and without voltage management.

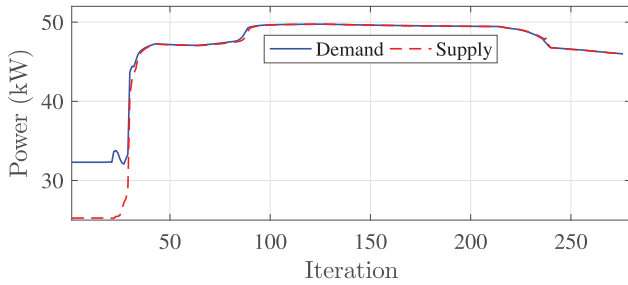


Fig. 9. Evolution of total demand and supply with voltage management.

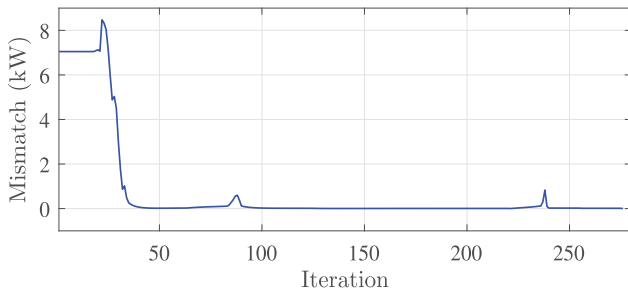


Fig. 10. Evolution of power imbalance Δp with voltage management.

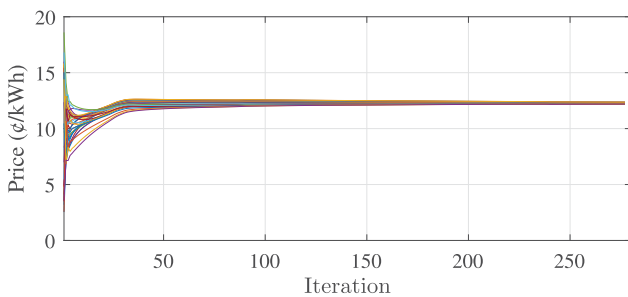


Fig. 11. Evolution of local prices with voltage management.

and the total supply and demand are balanced, as shown in Fig. 9. The evolution of local energy prices $\hat{\lambda}_l$ and the evolution of local estimation of mismatch power $\Delta \hat{p}_l$ with voltage management are shown in Fig. 11 and 12, respectively. It can be seen in Fig. 8 that voltage magnitudes fall within the specified operating

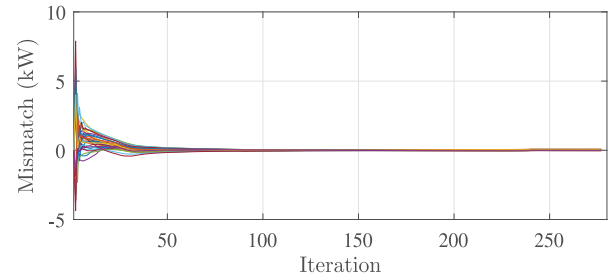


Fig. 12. Evolution estimation of power mismatch with voltage management.

limits of $\pm 5\%$ of 1 p.u. at all nodes when voltage management is considered in the energy pricing mechanism.

C. Two-Stage Approach

It is clear that algorithm takes a longer time to converge with voltage management ($k = 277$) as compared with the case without voltage management ($k = 187$). It is due to the inclusion of network voltage management strategy in the energy pricing mechanism. It is required to run voltage management algorithm, i.e., Algorithm 2, in each iteration of the proposed decentralized algorithm as shown in Fig. 1, and the convergence time is increased. As explained in Section V-A, it may not be practical to run the algorithm for a large number of iterations to reach a very precise solution in real-time applications. The increase in convergence time reduces the practicability of the algorithm for real-time implementation. We propose a two-stage approach for energy pricing considering these factors to increase the viability of the algorithm for real-time implementation. At the first stage, the energy pricing algorithm is executed without considering the voltage management until the equilibrium condition is reached. The intermediate block containing Algorithm 2 of Fig. 1 is skipped at this stage. All the node voltages are examined for possible violations at the beginning of the second stage. If there are no voltage violations in the network, then the solution from the first stage is the final solution. Otherwise, the algorithm proceeds to the second stage considering the voltage management strategy in the energy pricing mechanism and executed until the final equilibrium condition is achieved. The solution obtained from the first stage is taken as an initial input to the second stage, which helps in fast convergence of the second stage.

The evolution of the supply and demand curves in a two-stage approach is shown in Fig. 13. The final power balance condition obtained from the two-stage approach shown in Fig. 13 is the same as that obtained from the single-stage approach with voltage management, as shown in Fig. 9. The termination conditions are satisfied when $k = 277$ in a two-stage approach. As explained in Section IV-C, the proposed energy trading algorithm needs to be run at the beginning of each time period and the outcome is kept constant throughout that period. It means that the communication and computational facilities remain idle for the rest of the time during that period once the algorithm is converged. It makes possible to run the first stage of the

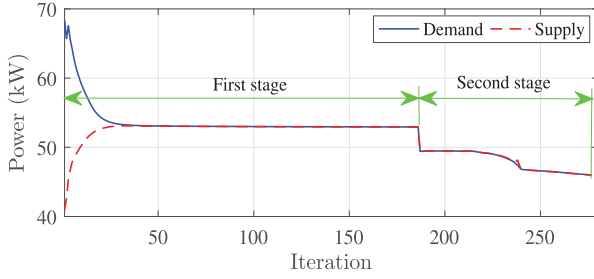


Fig. 13. Evolution of total demand and supply in a two-stage approach.

algorithm for the next period during the idle duration of the previous period. Practically, only the second stage has to be run in real time, which helps to achieve faster convergence and increase the practicability of the algorithm. In our case study, the second stage of the algorithm converges in $277 - 187 = 90$ iterations. This is the required number of iterations that need to be run in real time in a two-stage approach, whereas in a single-stage approach with voltage management, all 277 iterations need to be run in real time. Moreover, running the first-stage algorithm during the idle duration helps in effective utilization of available communication and computation facilities. Thus, the two-stage approach is more suitable for real implementation.

VI. CONCLUSION

In this article, we presented an energy pricing mechanism for decentralized local energy trading in microgrids with distributed generations and responsive demands considering the network voltage management. The local energy trading problem in a microgrid was formulated as an aggregated welfare maximization problem. A fully decentralized approach based on local communication with neighbors was proposed for energy pricing and voltage management. The proposed method was applied to a microgrid with the IEEE 33-node network structure. The convergence of the proposed method was verified via numerical results. The simulation results demonstrated that the proposed decentralized approach through the coordination among the agents establishes the supply–demand balance in the microgrid, decides the energy trading price, and maintains all the node voltages within the safe operating limits. The two-stage approach improved the convergence speed of the algorithm and increased the practicability of the algorithm for real-time implementation. This work can be extended by considering power losses and network charges in local energy trading as well as the effects of multiperiod optimization on real-time decision-making.

APPENDIX

A. CONVERGENCE ANALYSIS OF DECENTRALIZED ALGORITHM FOR LOCAL ENERGY TRADING

The updating rules of consensus-based decentralized algorithm (17) and (19) can be rewritten in the following form:

$$\hat{\lambda}^k = \mathbf{E}\hat{\lambda}^{k-1} + \rho\Delta\hat{\mathbf{p}}^{k-1} \quad (33)$$

$$\Delta\hat{\mathbf{p}}^k = \mathbf{E}\Delta\hat{\mathbf{p}}^{k-1} + \mathbf{u}^{k-1} \quad (34)$$

where $\hat{\lambda}$ and $\Delta\hat{\mathbf{p}}$ are the column vectors of $\hat{\lambda}_l$ and $\Delta\hat{p}_l$, respectively.

Writing (33) and (34) in a vector form

$$\begin{bmatrix} \hat{\lambda}^k \\ \Delta\hat{\mathbf{p}}^k \end{bmatrix} = \begin{bmatrix} \mathbf{E} & \rho\mathbf{I} \\ 0 & \mathbf{E} \end{bmatrix} \begin{bmatrix} \hat{\lambda}^{k-1} \\ \Delta\hat{\mathbf{p}}^{k-1} \end{bmatrix} + \begin{bmatrix} 0 \\ \mathbf{I} \end{bmatrix} \mathbf{u}^{k-1}. \quad (35)$$

The vector \mathbf{u}^{k-1} can be expressed in terms of $\hat{\lambda}^{k-1}$ and $\Delta\hat{\mathbf{p}}^{k-1}$ as follows:

$$\mathbf{u}^{k-1} = \tilde{\mathbf{B}} \left((\mathbf{I} - \mathbf{E})\hat{\lambda}^{k-1} - \rho\Delta\hat{\mathbf{p}}^{k-1} \right) \quad (36)$$

where $\tilde{\mathbf{B}} = \text{diag}([\tilde{b}_1, \tilde{b}_2, \dots, \tilde{b}_N])$, and

$$\begin{cases} 0 \leq \tilde{b}_l \leq \frac{1}{2a_l} & \forall l \in \mathcal{P} \\ 0 \leq \tilde{b}_l \leq \frac{1}{2\theta_l} & \forall l \in \mathcal{C} \end{cases}. \quad (37)$$

Now, (35) can be written as

$$\begin{bmatrix} \hat{\lambda}^k \\ \Delta\hat{\mathbf{p}}^k \end{bmatrix} = \begin{bmatrix} \mathbf{E} & \rho\mathbf{I} \\ (\tilde{\mathbf{B}}\mathbf{I} - \mathbf{E}) & (\mathbf{E} - \rho\tilde{\mathbf{B}}) \end{bmatrix} \begin{bmatrix} \hat{\lambda}^{k-1} \\ \Delta\hat{\mathbf{p}}^{k-1} \end{bmatrix}. \quad (38)$$

Define \mathbf{H} as

$$\mathbf{H} = \begin{bmatrix} \mathbf{E} & \rho\mathbf{I} \\ (\tilde{\mathbf{B}}\mathbf{I} - \mathbf{E}) & (\mathbf{E} - \rho\tilde{\mathbf{B}}) \end{bmatrix}. \quad (39)$$

We use the eigenvalue approach [29] to analyze the convergence properties. If ρ is chosen small enough to be neglected, \mathbf{H} has the repeated eigenvalue set of eigenvalues of \mathbf{E} , i.e.,

$$|\sigma\mathbf{I} - \mathbf{H}| = |\sigma\mathbf{I} - \mathbf{E}|^2. \quad (40)$$

As \mathbf{E} is a doubly stochastic matrix and irreducible, 1 is a simple eigenvalue of \mathbf{E} and the others have moduli smaller than 1. Thus, $\sigma_1 = \sigma_2 = 1$ is the double eigenvalue of \mathbf{H} , and the remaining eigenvalues have moduli smaller than 1. The eigenvalue 1 leads the evolution of consensus variables to a common value. It can be verified that $[\mathbf{1} \ 0]^T$ is the eigenvector of \mathbf{H} associated with eigenvalue $\sigma_1 = 1$, i.e.,

$$\begin{bmatrix} \mathbf{E} & \rho\mathbf{I} \\ (\tilde{\mathbf{B}}\mathbf{I} - \mathbf{E}) & (\mathbf{E} - \rho\tilde{\mathbf{B}}) \end{bmatrix} \begin{bmatrix} \mathbf{1} \\ 0 \end{bmatrix} = \begin{bmatrix} \mathbf{1} \\ 0 \end{bmatrix}. \quad (41)$$

As the eigenvalue 1 leads the algorithm to convergence, regardless of the value of ρ , $[\hat{\lambda}^k \ \Delta\hat{\mathbf{p}}^k]^T$ converges to span $[\mathbf{1} \ 0]^T$, i.e.,

$$\lim_{k \rightarrow \infty} \begin{bmatrix} \hat{\lambda}^k \\ \Delta\hat{\mathbf{p}}^k \end{bmatrix} = \begin{bmatrix} \lambda^* \\ 0 \end{bmatrix}. \quad (42)$$

B. CONVERGENCE ANALYSIS OF VOLTAGE MANAGEMENT ALGORITHM

The update rules in (29) can be generalized at any iteration r in a compact form as

$$\boldsymbol{\mu}^r = \mathbf{E}\boldsymbol{\mu}^{r-1}; \quad \bar{\boldsymbol{v}}^r = \mathbf{E}\bar{\boldsymbol{v}}^{r-1}; \quad \underline{\boldsymbol{v}}^r = \mathbf{E}\underline{\boldsymbol{v}}^{r-1} \quad (43)$$

where $\boldsymbol{\mu}$, $\bar{\boldsymbol{v}}$, and $\underline{\boldsymbol{v}}$ are the column vectors of μ_l , \bar{v}_l , and \underline{v}_l , respectively. After r iterations, the relation between the vectors

μ^r , $\bar{\nu}^r$, $\underline{\nu}^r$ can be expressed in terms of the initial vectors μ^0 , $\bar{\nu}^0$, $\underline{\nu}^0$, respectively, as

$$\mu^r = \mathbf{E}^r \mu^0; \bar{\nu}^r = \mathbf{E}^r \bar{\nu}^0; \underline{\nu}^r = \mathbf{E}^r \underline{\nu}^0. \quad (44)$$

As \mathbf{E} is doubly stochastic, non-negative, irreducible, and aperiodic matrix [30], $\lim_{r \rightarrow \infty} \mathbf{E}^r$ converges to a rank-one deterministic matrix. This ensures that variables in (44) converge to the unique solutions $\lim_{r \rightarrow \infty} \mu_l^r = \zeta_l \sum_{l=1}^N \mu_l^0$, $\lim_{r \rightarrow \infty} \bar{\nu}_l^r = \zeta_l \sum_{l=1}^N \bar{\nu}_l^0$, and $\lim_{r \rightarrow \infty} \underline{\nu}_l^r = \zeta_l \sum_{l=1}^N \underline{\nu}_l^0$, where ζ_l is the l th element of the unit eigenvector corresponding to eigenvalue 1 of the matrix \mathbf{E} and satisfies $\sum_{l=1}^N \zeta_l = 1$ and $\zeta_l > 0$ [30]. If $\sum_{l=1}^N \mu_l^0 \geq 0$, then $\lim_{r \rightarrow \infty} \mu_l^r \geq 0$, and if $\sum_{l=1}^N \mu_l^0 < 0$, then $\lim_{r \rightarrow \infty} \mu_l^r < 0$. Therefore, for $\lim_{r \rightarrow \infty} \mu_l^r \geq 0$, as $r \rightarrow \infty$

$$\lim_{r \rightarrow \infty} \frac{\mu_l^r}{\bar{\nu}_l^r} = \frac{\zeta_l \sum_{l=1}^N \mu_l^0}{\zeta_l \sum_{l=1}^N \bar{\nu}_l^0} = \frac{\sum_{l=1}^N \mu_l^0}{\sum_{l=1}^N \bar{\nu}_l^0} \quad (45)$$

$$\eta_l = \lim_{r \rightarrow \infty} \eta_l^r = \frac{\sum_{l=1}^N \mu_l^0}{\sum_{l=1}^N \bar{\nu}_l^0} \quad \forall l. \quad (46)$$

Similarly, for $\lim_{r \rightarrow \infty} \mu_l^r < 0$, as $r \rightarrow \infty$

$$\eta_l = \lim_{r \rightarrow \infty} \eta_l^r = \frac{\sum_{l=1}^N \mu_l^0}{\sum_{l=1}^N \underline{\nu}_l^0} \quad \forall l. \quad (47)$$

REFERENCES

- [1] A. Ramos, C. D. Jonghe, V. Gmez, and R. Belmans, "Realizing the smart grid's potential: Defining local markets for flexibility," *Utilities Policy*, vol. 40, pp. 26–35, 2016.
- [2] S. Abapour, K. Zare, and B. Mohammadi-Ivatloo, "Dynamic planning of distributed generation units in active distribution network," *IET Gener., Transmiss. Distrib.*, vol. 9, no. 12, pp. 1455–1463, 2015.
- [3] M. Khorasany, Y. Mishra, and G. Ledwich, "Market framework for local energy trading: A review of potential designs and market clearing approaches," *IET Gener., Transmiss. Distrib.*, vol. 12, pp. 5899–5908(9), Dec. 2018.
- [4] A. M. Jadhav and N. R. Patne, "Priority-based energy scheduling in a smart distributed network with multiple microgrids," *IEEE Trans. Ind. Informat.*, vol. 13, no. 6, pp. 3134–3143, Dec. 2017.
- [5] M. Pilz and L. Al-Fagih, "Recent advances in local energy trading in the smart grid based on game-theoretic approaches," *IEEE Trans. Smart Grid*, vol. 10, no. 2, pp. 1363–1371, Mar. 2019.
- [6] J. Guerrero, A. C. Chapman, and G. Verbi, "Decentralized p2p energy trading under network constraints in a low-voltage network," *IEEE Trans. Smart Grid*, vol. 10, no. 5, pp. 5163–5173, Sep. 2019.
- [7] A. Paudel, K. Chaudhari, C. Long, and H. B. Gooi, "Peer-to-peer energy trading in a prosumer-based community microgrid: A game-theoretic model," *IEEE Trans. Ind. Electron.*, vol. 66, no. 8, pp. 6087–6097, Aug. 2019.
- [8] Y. Wu, X. Tan, L. Qian, D. H. K. Tsang, W. Song, and L. Yu, "Optimal pricing and energy scheduling for hybrid energy trading market in future smart grid," *IEEE Trans. Ind. Informat.*, vol. 11, no. 6, pp. 1585–1596, Dec. 2015.
- [9] N. Li, "A market mechanism for electric distribution networks," in *Proc. 54th IEEE Conf. Decis. Control*, Dec. 2015, pp. 2276–2282.
- [10] W. H. Kresting, *Distribution System Modeling and Analysis*. New York, NY, USA: CRC Press, 2001.
- [11] Z. Li, L. Chen, and G. Nan, "Small-scale renewable energy source trading: A contract theory approach," *IEEE Trans. Ind. Informat.*, vol. 14, no. 4, pp. 1491–1500, Apr. 2018.
- [12] C. P. Mediwaththe, M. Shaw, S. K. Halgamuge, D. Smith, and P. M. Scott, "An incentive-compatible energy trading framework for neighborhood area networks with shared energy storage," *IEEE Trans. Sustain. Energy*, vol. 11, no. 1, pp. 467–476, Jan. 2020.
- [13] T. Chen and W. Su, "Indirect customer-to-customer energy trading with reinforcement learning," *IEEE Trans. Smart Grid*, vol. 10, no. 4, pp. 4338–4348, Jul. 2019.
- [14] L. Park, S. Jeong, J. Kim, and S. Cho, "Joint geometric unsupervised learning and truthful auction for local energy market," *IEEE Trans. Ind. Electron.*, vol. 66, no. 2, pp. 1499–1508, Feb. 2019.
- [15] G. Sun, F. Zhang, D. Liao, H. Yu, X. Du, and M. Guizani, "Optimal energy trading for plug-in hybrid electric vehicles based on fog computing," *IEEE Internet Things J.*, vol. 6, no. 2, pp. 2309–2324, Apr. 2019.
- [16] T. Morstyn, A. Teytelboym, and M. D. McCulloch, "Bilateral contract networks for peer-to-peer energy trading," *IEEE Trans. Smart Grid*, vol. 10, no. 2, pp. 2026–2035, Mar. 2019.
- [17] W. Tushar *et al.*, "Grid influenced peer-to-peer energy trading," *IEEE Trans. Smart Grid*, vol. 11, no. 2, pp. 1407–1418, Mar. 2020.
- [18] B. Zhang, C. Jiang, J. Yu, and Z. Han, "A contract game for direct energy trading in smart grid," *IEEE Trans. Smart Grid*, vol. 9, no. 4, pp. 2873–2884, Jul. 2018.
- [19] E. Dall'Anese, H. Zhu, and G. B. Giannakis, "Distributed optimal power flow for smart microgrids," *IEEE Trans. Smart Grid*, vol. 4, no. 3, pp. 1464–1475, Sep. 2013.
- [20] W. Shi, X. Xie, C. Chu, and R. Gadh, "Distributed optimal energy management in microgrids," *IEEE Trans. Smart Grid*, vol. 6, no. 3, pp. 1137–1146, May 2015.
- [21] T. Lu, Z. Wang, Q. Ai, and W. Lee, "Interactive model for energy management of clustered microgrids," *IEEE Trans. Ind. Appl.*, vol. 53, no. 3, pp. 1739–1750, May 2017.
- [22] P. Samadi, A. H. Mohsenian-Rad, R. Schober, V. W. S. Wong, and J. Jatskevich, "Optimal real-time pricing algorithm based on utility maximization for smart grid," in *Proc. 1st IEEE Int. Conf. Smart Grid Commun.*, Oct. 2010, pp. 415–420.
- [23] M. Khorasany, Y. Mishra, and G. Ledwich, "A decentralised bilateral energy trading system for peer-to-peer electricity markets," *IEEE Trans. Ind. Electron.*, vol. 67, no. 6, pp. 4646–4657, Jun. 2020.
- [24] M. E. Baran and F. F. Wu, "Network reconfiguration in distribution systems for loss reduction and load balancing," *IEEE Trans. Power Del.*, vol. 4, no. 2, pp. 1401–1407, Apr. 1989.
- [25] M. Farivar, L. Chen, and S. Low, "Equilibrium and dynamics of local voltage control in distribution systems," in *Proc. 52nd IEEE Conf. Decis. Control*, Dec. 2013, pp. 4329–4334.
- [26] S. Boyd, N. Parikh, E. Chu, B. Peleato, and J. Eckstein, "Distributed optimization and statistical learning via the alternating direction method of multipliers," *Found. Trends Mach. Learn.*, vol. 3, no. 1, pp. 1–122, Jan. 2011.
- [27] R. Olfati-Saber, J. A. Fax, and R. M. Murray, "Consensus and cooperation in networked multi-agent systems," *Proc. IEEE*, vol. 95, no. 1, pp. 215–233, Jan. 2007.
- [28] X. Wang *et al.*, "Optimal voltage regulation for distribution networks with multi-microgrids," *Appl. Energy*, vol. 210, pp. 1027–1036, 2018.
- [29] K. Cai and H. Ishii, "Average consensus on general digraphs," in *Proc. 50th IEEE Conf. Decis. Control Eur. Control Conf.*, 2011, pp. 1956–1961.
- [30] R. A. Horn and C. R. Johnson, *Matrix Analysis*. Cambridge, U.K.: Cambridge Univ. Press, 1985.